Closing Fri: 2.1, 2.2, 2.3

Closing next Tue: 2.5-6

Closing next Fri: 2.7, 2.7-8

Today: Finish 2.3, start 2.5.

Entry Task: Find the limits

1. $\lim_{x\to 0} \frac{x^2+6x+5}{x+1} =$

2. $\lim_{x\to -1} \frac{x^2+6x+5}{x+1} =$

3. $\lim_{x\to 0^+} \frac{e^x - \sin\left(x+\frac{\pi}{2}\right)+3}{x(x-4)} =$

4. $\lim_{x\to 2} \frac{\frac{1}{x+1}-\frac{1}{3}}{x-2} =$

5. $\lim_{h \to 0} \frac{(6+h)^2 - 36}{h} =$

6. $\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2} =$

Squeeze Thm:

If the following hold:

(1) $g(x) \leq f(x) \leq h(x)$ near $x = a$

(2) $\lim_{x \to a} g(x) = L$ and $\lim_{x \to a} h(x) = L$

then

$$\lim_{x \to a} f(x) = L$$

Example: Find

$$\lim_{x \to 0} x^2 \cos\left(\frac{10}{x}\right) =$$

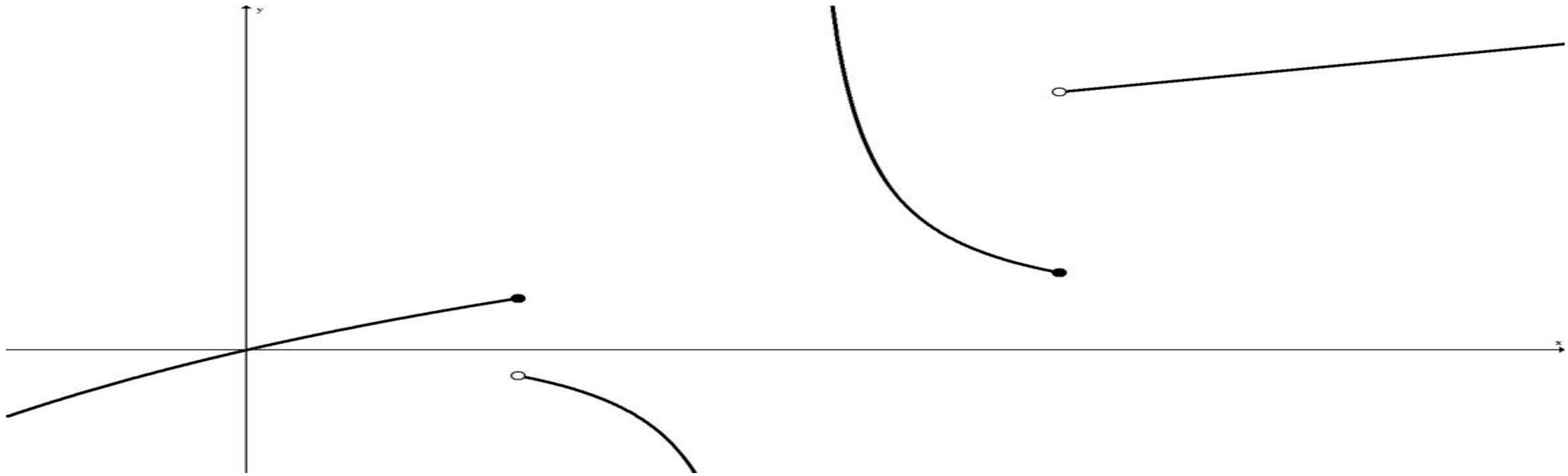
Multipart functions (again):

Example: Find the limits

$$f(x) = \begin{cases} \dfrac{12x}{x+5} & \text{, if } x \le 1; \\ \dfrac{x}{x-2} & \text{, if } 1 < x \le 3 \text{ and } x \ne 2; \\ \dfrac{x^2+4x-21}{x-3} & \text{, if } x > 3. \end{cases}$$

1. $\lim_{x \to 1^-} f(x)$
2. $\lim_{x \to 1^+} f(x)$
3. $\lim_{x \to 2^-} f(x)$
4. $\lim_{x \to 2^+} f(x)$
5. $\lim_{x \to 3^-} f(x)$
6. $\lim_{x \to 3^+} f(x)$

Here is a picture of f(x)



2.5 Continuity

A function, $f(x)$, is **continuous at x = a** if

$$\lim_{x \to a} f(x) = f(a)$$

this implies three things

(1). f(a) is defined,

(2). $\lim_{x \to a} f(x)$ exists and is finite, and

(3). (1) and (2) are the same!

Continuous from the left

$$\lim_{x \to a^-} f(x) = f(a)$$

Continuous from the right

$$\lim_{x \to a^+} f(x) = f(a)$$

Casually, we might say a function is continuous at *x = a* if you can draw the graph across *x = a* point without picking up your pencil.

The “standard” precalculus functions are **continuous everywhere they are defined**:

polynomials → defined everywhere
sin(x), cos(x) → defined everywhere
e^x → defined everywhere
odd roots → defined everywhere
$\tan^{-1}(x)$ → defined everywhere
Rational Functions → for denom ≠ 0
Even Roots → under radical ≥ 0
ln(x) → for x > 0
tan(x) → not at $x = \pm k\pi/2$
$\sin^{-1}(x)$, $\cos^{-1}(x)$ → for $-1 \le x \le 1$

Example:

$$g(x) = \begin{cases} 8 - x^2 & , \text{if } x < 0; \\ 2 & , \text{if } 0 \leq x < 5; \\ 0 & , \text{if } x = 5; \\ 7 - x & , \text{if } x > 5. \end{cases}$$

Does g(x) have any discontinuities?

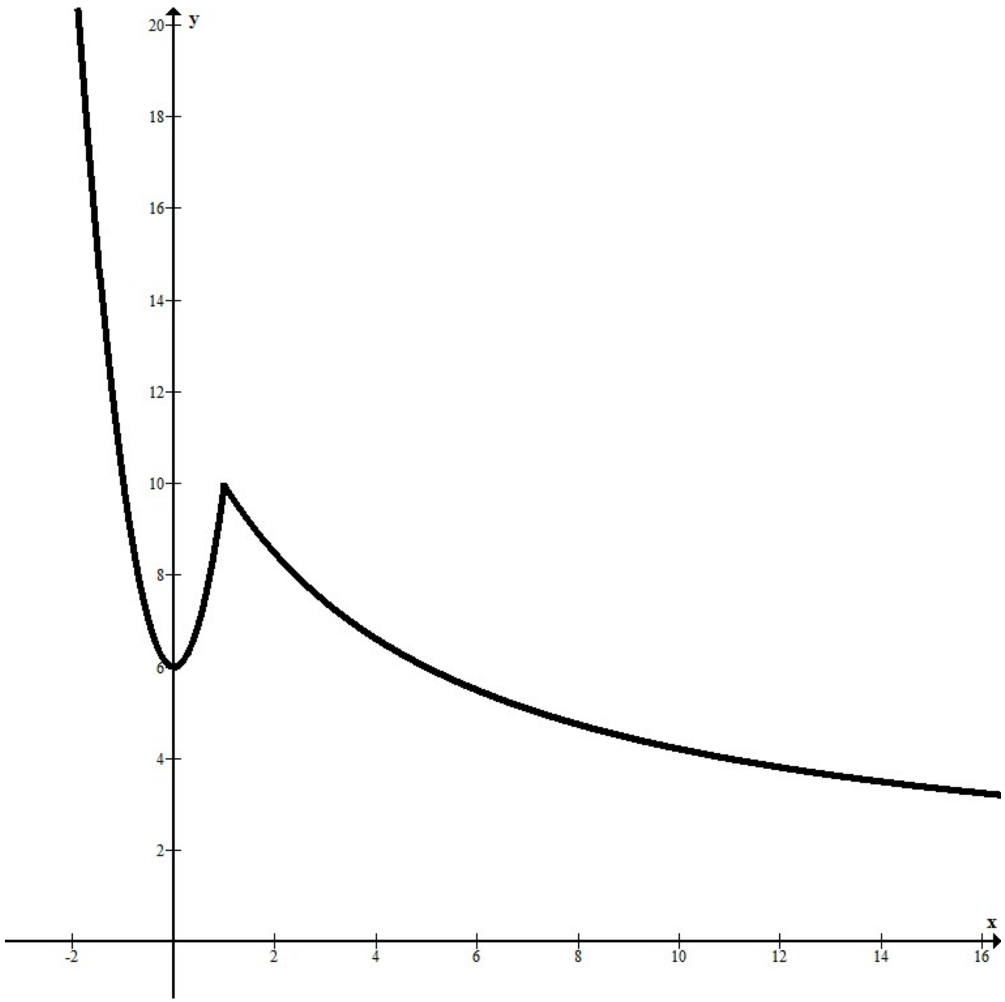
If so, where?

Example:

$$h(x) = \begin{cases} ax^2 + 6 & , \text{if } x < 1; \\ b & , \text{if } x = 1; \\ \dfrac{x+49}{x+a} & , \text{if } x > 1. \end{cases}$$

Find the values of *a* and *b* that will make $h(x)$continuous *everywhere*.

h(x)



Theorem:

If $f(x)$is continuous at $x = b$, and

$$\lim_{x \to a} g(x) = b$$

then

$$\lim_{x \to a} f\left(g(x)\right) = f(b).$$

Example:

Find

$$\lim_{x \to 9} \ln\left(\frac{\sqrt{x} - 3}{x - 9}\right)$$